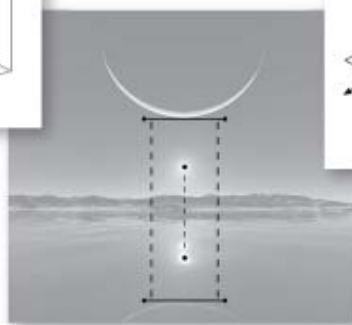
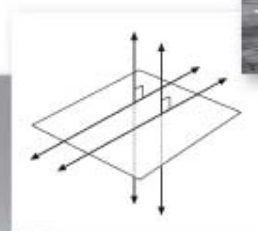
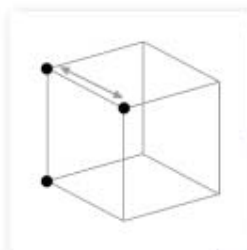




# Space Geometry

## Grade 11 Teacher's Guide

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## Introduction

This unit leads students to explore concepts in space geometry. It accompanies the active learning Online Teacher Training materials. You and your students will use Active Learning techniques in each lesson and some of them may be new. Have patience. It takes time for both you and your students to adapt to new ways of teaching and learning.

The students will be in small groups solving problems together for much of their classes. They will use physical objects to model space, including an x, y, z axis structure. The x, y, z axis structure was designed especially for use with this unit and built by the Teaching Materials Department.

You are the teacher. You know your student's interests, capabilities and limits best. You may wish to try the activities shown here and adapt them as needed for specific needs. For some of your classes, these may work well, for others you may need to make a few adjustments based on your students' abilities and interests. There are options given if activities take longer than expected. The student worksheets and activities are printed in the Appendices of this guide. In the homework section of each lesson, more problems are suggested, which can be found in Appendix C.

Because there are more homework assignments than are in the text, you should coordinate with other instructors at your grade level to manage the students' homework load. Students may feel overwhelmed if they receive homework from all their teachers on the same day. The lessons will follow the regular curriculum map for the unit shown below. The material will be presented in many ways besides lecture.

For some lessons videos are available in the Online Training. These lessons are indicated in this guide with: [VIDEO].

## Outline for Unit Five: Space Geometry

### Topics in Week 1

- Definition of plane.
- Number of points needed to define a line.
- First postulate:
  - a) A straight line includes at least two different points. A straight line is completely defined by any two points on it.
  - b) A plane includes at least three points not on the same line or a plane is completely defined by three points not on the same line.
- Second postulate: An infinite number of planes go through two points.
- Theorem 1: Any line (L) and a point outside it fall on one plane.
- Theorem 2: Two intersecting lines make a plane.
- Theorem 3: If two planes share a point, they also share a line.
- Exercises and problems, section 1.

### Topics in Week 2

- Definition of space.
- Postulate 3: A space is specified as four points not on a straight line.
- Lines and planes in space:
  - a) Relationship between a plane and a plane.
  - b) Relationship between a line and a plane.
- Conclusion: If a line is parallel to a plane, it does not have to be parallel to every line in it.
- Definition: Line (L) is perpendicular to plane (X) if line L is perpendicular on all lines on plane X that are located at the point of intersection between line L and plane X.
- Theorem 4: A line that is perpendicular on two non-parallel lines on a plane is perpendicular on the plane.
- Conclusion: A line which is parallel to another line which is perpendicular to a plane is also perpendicular to the same plane.
  - c) Relationship between two planes.
- Coordinates in three-dimensional space (finding coordinates of a point).
- Exercises and problems, Section 2.

## Topics in Week 3

- Coordinates in three-dimensional space (Drawing a point- distance between two points- coordinates of the midpoint of the distance between two points).
- Perpendicular Projections:
  - a) Projection of a point on a plane.
  - b) Projection of a line on a plane.
- Theorem: The projection of a bent line that is parallel to a line in a plane is parallel to that line.
- Converse of the theorem: A bent line is perpendicular to a plane if its projection is perpendicular to the plane.
- Dihedral Angles (straight angle for a dihedral angles- Measuring Dihedral Angles)
- Exercises and problems, Section 3
- General exercises

## Active Learning Techniques included in this module:

- Concept Mapping
- Journaling
- Inquiry
- Cooperative Learning
- Problem-based Learning
- Project-based Learning
- Socratic Questioning
- Modeling

## A Brief Overview of the Lessons in Unit 5

<b>Title</b>	<b>Objectives</b>	<b>Brief Description of the Lesson</b>	<b>Active Learning Techniques</b>
Definition of a Plane	<ul style="list-style-type: none"> <li>• To recognize a plane.</li> <li>• To define a plane.</li> </ul>	Students write their own definition of a plane and make concept maps with words provided by the teacher.	<ul style="list-style-type: none"> <li>• Concept Mapping</li> <li>• Journaling</li> <li>• Cooperative Learning</li> <li>• Brainstorming</li> <li>• Inquiry</li> </ul>
Postulates Related to Lines and Planes	<ul style="list-style-type: none"> <li>• To conclude the postulates related to the plane and the line.</li> </ul>	Students work in small groups with concrete materials to show that Postulates 1 and 2 are true. The teacher guides students to the postulates using Socratic Questioning.	<ul style="list-style-type: none"> <li>• Cooperative Learning</li> <li>• Socratic Questioning</li> </ul>
Theorem 1: Line and Point Define a Plane	<ul style="list-style-type: none"> <li>• To prove Theorem 1.</li> <li>• To solve problems related to the theorem.</li> </ul>	Students work in small groups with concrete materials to model the theorem, then solve a real-life exercises.	<ul style="list-style-type: none"> <li>• Cooperative Learning</li> <li>• Problem-based Learning</li> </ul>
Theorem 2: Two Lines Define a Plane	<ul style="list-style-type: none"> <li>• To prove Theorem 2.</li> <li>• To clarify the result related to Theorem 2.</li> </ul>	In small groups students model lines and planes with kabob sticks and paper. Students try to prove the theorem with hints from the teacher.	Cooperative Learning
Theorem 3: If Two Planes Share a Point They Share a Line.	<ul style="list-style-type: none"> <li>• To prove Theorem 3.</li> <li>• To solve problems related to the Theorems 1, 2 and 3</li> </ul>	Model Theorem 3 with concrete materials. Students work in pairs to solve problems on a worksheet.	<ul style="list-style-type: none"> <li>• Cooperative Learning</li> <li>• Problem-based Learning</li> <li>• Brainstorming</li> </ul>
Postulate 3: Define Space	<ul style="list-style-type: none"> <li>• To conclude/prove Postulate 3</li> </ul>	Students create space filling models with kabob sticks. They conclude they cannot make a three-dimensional shape with 3 vertices.	<ul style="list-style-type: none"> <li>• Cooperative Learning</li> <li>• Modeling</li> <li>• Brainstorming</li> </ul>



<b>Title</b>	<b>Objectives</b>	<b>Brief Description of the Lesson</b>	<b>Active Learning Techniques</b>
Lines and Planes in Space	<ul style="list-style-type: none"> <li>• To conclude the relationship between two straight lines.</li> <li>• To conclude the relationships between a line and a plane.</li> </ul>	Students model intersecting lines as well as intersections of lines and planes with concrete materials.	Cooperative Learning
Theorem 4: A Line Perpendicular to a Plane.	<ul style="list-style-type: none"> <li>• To explain Theorem 4, page 19.</li> <li>• To prove the theorem.</li> <li>• To explain the result related to the theorem.</li> </ul>	Students model the theorem using the classroom, then the teacher proves the theorem. Students work on an exercise individually to practice using the theorem.	<ul style="list-style-type: none"> <li>• Cooperative Learning</li> <li>• Inquiry</li> </ul>
Relationship between Two Planes in Space.	<ul style="list-style-type: none"> <li>• To conclude the relationship between plane and plane.</li> <li>• To solve problems on lines and planes in space.</li> </ul>	Students model the intersection of the planes using concrete materials, then solve problems on lines and planes in space.	<ul style="list-style-type: none"> <li>• Cooperative Learning</li> <li>• Modeling</li> </ul>
Coordinates of a Point in Three-dimensional Space, Part 1.	<ul style="list-style-type: none"> <li>• To recognize the coordinate system.</li> <li>• To describe a point in three-dimensional space using the x, y, z coordinate system.</li> </ul>	Students do the activity which is in the book as a game to help them recognize the 3 axes needed to determine the position of a point in space. Students describe a point in the room using three-dimensional coordinates. The teacher explains x, y, z coordinate system.	<ul style="list-style-type: none"> <li>• Cooperative Learning</li> <li>• Game</li> </ul>
Coordinates of a Point in Three-dimensional Space, Part 2.	<ul style="list-style-type: none"> <li>• To draw a given (x ,y ,z) point in three-dimensional space.</li> </ul>	The teacher demonstrates how to use x, y, z coordinates to draw a point in three-dimensional space. Students practice drawing points using models if necessary.	Modeling

<b>Title</b>	<b>Objectives</b>	<b>Brief Description of the Lesson</b>	<b>Active Learning Techniques</b>
Distance between Two Points in Three-Dimensional Space.	<ul style="list-style-type: none"> <li>• To find the distance between two points in three-dimensional space.</li> </ul>	<p>Students measure the distance between two points in three-dimensional space using a tissue box. Then they calculate the distance using the distance formula.</p> <p>Next, students compare between the measured and calculated distances.</p>	<ul style="list-style-type: none"> <li>• Inquiry</li> <li>• Cooperative Learning</li> </ul>
Midpoints and Projections	<ul style="list-style-type: none"> <li>• To find the midpoint in three-dimensional space.</li> <li>• To define the term “projection” as it is used in geometry.</li> </ul>	<p>Students use a model to measure and calculate the midpoint of a line segment in three-dimensional space.</p> <p>Then students compare between the measured and calculated distances.</p> <p>Students draw projections. The teacher extends their definitions.</p>	<ul style="list-style-type: none"> <li>• Inquiry</li> <li>• Cooperative Learning</li> <li>• Discussion</li> </ul>
Perpendicular Projection	<ul style="list-style-type: none"> <li>• To prove Theorem 4.</li> <li>• To recognize the converse of the theorem.</li> </ul>	<p>In groups, students represent the theorem by using given materials. The teacher provides opportunities for students to prove the theorem and then discusses the proof with the students.</p> <p>The teacher writes the statement of the converse of the theorem on the board and asks the students (in groups) to demonstrate the converse of the theorem by using the same materials. The teacher assigns proof of the converse of the theorem as homework.</p>	<ul style="list-style-type: none"> <li>• Cooperative Learning</li> </ul>
Dihedral Angles	<ul style="list-style-type: none"> <li>• To define dihedral angle.</li> <li>• To measure dihedral angles.</li> </ul>	<p>The teacher revises the proofs of converse of the theorem students did as homework.</p> <p>Students provide real-life examples of dihedral angles. Students add the examples to their concept map. Students practice naming dihedral angles with help from the teacher. Students attempt to measure dihedral angles on regular and irregular shapes. The teacher explains how to measure dihedral angles on regular shapes.</p>	<ul style="list-style-type: none"> <li>• Cooperative Learning</li> <li>• Problem-based Learning</li> <li>• Concept Mapping</li> </ul>

## **Some Practical Advice for Implementing Active Learning in the Classroom**

### **Seating Arrangements for Cooperative Learning**

There are two easy ways to group students in cooperative groups: pairs and quads. Pairs are easily created by having students already sitting together at a table work together. Quads, which are groups of 4, are easily formed by having two students turn around to work with the two students behind them. Brief activities or simple and short questions should be done in pairs. Quads are best for more difficult problems or longer activities.

To facilitate assigning group roles to students, the desks should be numbered within a group. The desks don't need physical labels on them; students just need to know that the seat in front of the room nearest the door will be desk 1, the seat closest to the windows in front will be desk 2, etc.

Roles can be assigned to students by seat number. For example: number 1 can be the recorder, number 2 can be group leader, number 3 can be group quizzier, and number 4 can be the materials manager (draw diagrams for the group).

Teachers should arrange the seating so that students of different abilities are sitting in a group of either pairs or quads. For example, you could put an A student in seat 4, a B in seat 2, a C student in seat 3 and a D student in seat 1. Research has shown that Cooperative Learning works best if good students and struggling students work together. Working in groups may be new for students and can lead to some extra socializing in class for a few days. The novelty of sitting in groups will soon wear off in a few days and students will quiet down. It is important to stress that students should be talking about the math topic for the day, not other concerns. Taking points away from students that are socializing is an effective way to keep students on task.

### **The Five Essential Elements of Cooperative Learning**

There is a difference between Cooperative Learning and group work. Cooperative Learning IS NOT just putting students in groups to work together. At least four of the five elements listed below must be present in order for an activity to be considered as Cooperative Learning.

#### **Group Dynamics**

At the end of a group session, the group takes a little time to discuss how well they worked together. The group learns to recognize whether it is functioning well or poorly. The group takes action to improve cooperation or participation between members if needed.

## **Individual Accountability**

Each group member is responsible for their own understanding and for their responsibility to the group. Assigning roles to group members gives each student a responsibility. Group members may be assessed independently and then as a group with bonus points given to students who do better on a group quiz.

## **Positive Interdependence**

To be Cooperative Learning, there must be a reason for the group to work together. In math and science classes this is easily achieved by having students share materials. Other ideas for creating positive interdependence include: using problems that are too difficult for one student to solve alone, creating projects where each student is responsible for part of the work, or having each student learn a piece of information and teach it to the group.

## **Social Skills**

Learning how to work with other people is an important work place skill. It is an important part of Cooperative Learning as well. Students need to learn to disagree politely, do their fair share of work, communicate their ideas, and appreciate the contribution from other group members.

## **Face-to-Face Interaction**

Students need to be seated close together to work effectively as a cooperative group. The smaller the group the more easily members can interact with each other. Pairs are an ideal way to provide maximum face to face interaction. It's hard for groups of more than four to include everyone. Groups in this unit are four or two. If there is not an even number of students, create a group of three. Don't let students work alone.

## Week 1 Day 1 Topic: Definition of a Plane Time (35 minutes)

The Active Learning techniques included in this lesson are:

- Journaling
- Cooperative Learning - Brainstorming
- Concept Mapping
- Inquiry

### Journaling

Journaling is different than taking notes. Notes are what the teacher and text say. Journaling is the student's own words and thoughts. It is important to explain to the students that their journal entries should be in their own words. Their first entries on a topic do not need to be perfect.

Since Journaling is different, students should use a separate part of their notebook for journaling. The last 10-20 pages of the student's math notebook are a good place for them to write their journal entries. Putting journal entries in the notebook ensures students will have their journals with them in class.

After students are done writing, it is important to select a student at random to read their answer to the class. Choosing someone at random encourages all students to do the activity so they are prepared to read if selected.

In this lesson, it is important for students to recognize the difference between the first definition they wrote and the second definition. Students can write the difference between the two definitions so they remember what they learned.

### Cooperative Learning

To provide individual accountability in Cooperative Learning each student should have an assigned role. In this task possible roles could be:

- **Leader**, keeps the group on task and makes sure everyone is doing their job.
- **Recorder**, writes down brainstorming ideas, and other records of the groups' thinking.
- **Quizzer**, makes sure everyone in the group understands what is being discussed or concluded.
- **Materials Manager**, gathers all materials for the group to use and puts them away; ensures all group members take turns handling the materials.

A Cooperative Learning Roles Chart is included in Appendix A to aid in assigning student roles. As students adjust to working in a group, conflicts between group members may arise. At first you should help the group solve their problems. As students gain more experience working together, they should be able to resolve their own problems.

## Brainstorming

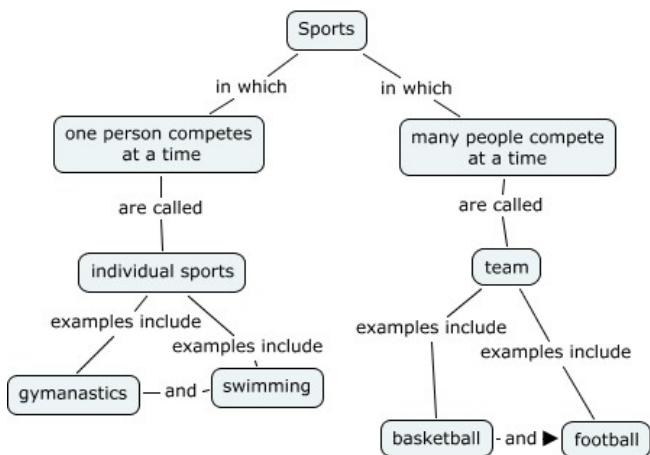
In this lesson brainstorming is done in pairs for 1 or 2 minutes. Then the teacher should write all student ideas on the board and discuss them as a class. In brainstorming all ideas are accepted without criticizing. Sorting through the ideas happens after the brainstorming session.

## Inquiry

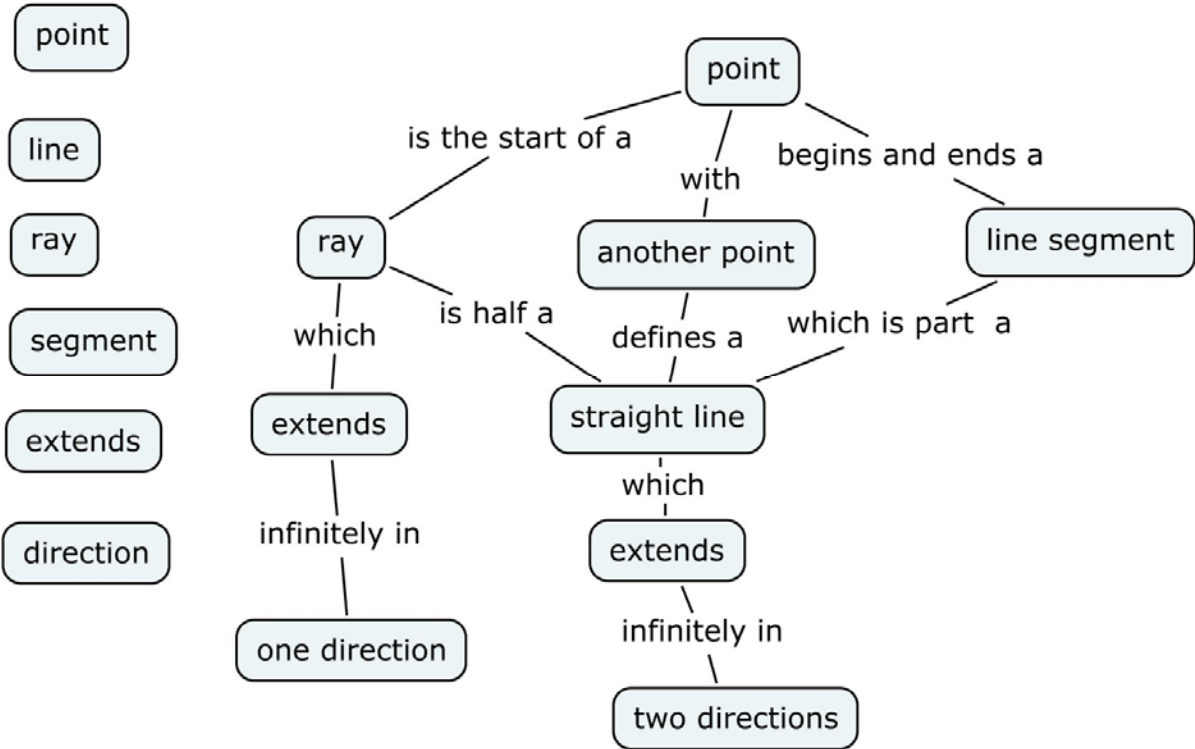
The value of Inquiry is letting students form their own ideas about the concepts. Do not tell the students about the intersection of a line and sphere or a line and a plane until after they have written down their own prediction. Ask a few students to read their predictions to the class. Select students who have different predictions. Ask them how they would show their prediction is correct.

## Concept Mapping

While students may have constructed concept maps in other courses they may need some help to make a concept map using math terms. You may want to refresh their memory by constructing a concept map of a familiar topic on the board. An example of a simple map is given below.



After the students map some simple concepts, they should be given some math terms to map. A sample list of terms and one possible map are given below:



- point
- line
- ray
- segment
- extends
- direction

## Activity Outline

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>• To recognize a plane.</li> <li>• To define a plane.</li> </ul>	<ol style="list-style-type: none"> <li>1. Briefly review the definitions of the following terms: point, ray, straight segment, straight line. <b>(Discussion)</b></li> <li>2. Students in groups brainstorm real life examples of a plane and a non plane. [VIDEO] Plane examples: ceiling, door or floor. Non-plane examples: ball, box or table. <b>(Cooperative Learning).</b></li> <li>3. Assessment: Students make a concept map with the definitions above (point, line, segment, ray, plane). The concept map may not look like the one listed for this lesson. There are many ways to represent the relationship between these terms. Students should leave room on their paper to add terms to their concept map on Day 4. <b>(Cooperative Learning, Pairs, Concept Mapping)</b></li> <li>4. Students individually write their own definitions of a plane. <b>(Journaling)</b></li> <li>5. The teacher asks students individually to predict what will happen if a line is placed in a ball (non-plane) and in the surface of the table (plane).  Students write answers on their notebooks. Then the teacher asks students to demonstrate different predictions using a ball and the surface of the table. Then the teacher gives the exact definition of the plane.</li> <li>6. Assessment <b>(Journaling).</b> The teacher asks the students individually to again define a plane according to what they have learned.</li> </ol>	<ul style="list-style-type: none"> <li>• Pictures</li> <li>• Ball</li> <li>• Table</li> <li>• Pen</li> </ul>	<p>Using the photos from Appendix B, identify points, lines, rays, segments and planes. (The teacher may wish to remove the labels on the photos before copying.)</p>	<p>Project: The teacher asks students to:</p> <ol style="list-style-type: none"> <li>1. Provide several photos of real life objects that represent a line, plane, ray, straight segment, and straight plane.</li> <li>2. EX 2, page 52</li> <li>3. Think: How many straight lines pass through a given point?</li> </ol>



## Week 1 Day 2 Topic: Postulates Related to Lines and Planes

### Discuss Previous Homework

While this is not an Active Learning technique, it is a good idea for students to prepare for class by thinking about the questions given in the previous lesson. Students' replies to the question are a measure of their knowledge. By assessing answers to these questions, teachers can easily find out the level of students' thinking, what they already know and what they don't understand.

If students are thinking about the new material the night before the lesson, the transition to new ideas is easier and faster for both the students and the teacher.

### Socratic Questioning

It is important not to list the postulates on the board before students discuss the activities they do in class. Socratic Questioning encourages students to use the evidence they gathered in previous activities to write the postulates themselves. They can look up the exact wording of the postulates at the end of the lesson.

Some sample Socratic Questions for this lesson might be:

- Why did you all draw the same line when there were two points on the board?
- Is it possible to draw a plane with four points?
- Why are three points sufficient?
- Are there any other ways to define a plane?

Later you can ask:

Can you find other examples besides those given in your book to illustrate Postulate 2? (For example: door on hinges, revolving door at hotel)

## Cooperative Learning

### Roles

Remember to assign roles to students in their groups. Changing roles each time the group forms keeps students interested in participating. The same roles can be used from day to day, but new students should fill each role. The four roles used in Day 1 would also work for this activity:

- **Leader**, keeps the group on task and makes sure everyone is doing their job.
- **Recorder**, writes down brainstorming ideas, and other records of the groups' thinking.
- **Quizzer**, makes sure everyone in the group understands what is being discussed or concluded.
- **Materials Manager**, gathers all materials for the group to use and puts them away, ensures all group members get a turn to use the materials.

To make determining roles easier you can post a chart in the front of the classroom that looks like this:

<b>Day</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>A</b>	Leader	Materials Manager	Quizzer	Recorder
<b>B</b>	Recorder	Leader	Materials Manager	Quizzer
<b>C</b>	Quizzer	Recorder	Leader	Materials Manager
<b>D</b>	Materials Manager	Quizzer	Recorder	Leader

The numbers on the top are student seat numbers in the group. They should know their seat number from the first day. Students should stay in the same seat from day to day. The teacher can choose at random whether it is an A, B, C or D day. The students can see on the chart which seat number has a particular role. In Appendix A this chart is provided in a larger form for copying to an overhead or posting in the classroom.

### Group Dynamics

As students become accustomed to working in groups it is important to focus on group dynamics and social skills. Give time at the end of the lesson for groups to evaluate how well they worked together. Groups should discuss:

- Did they do a quality work?
- Did all group members contribute equally? Or did one member do most of the work?
- Were everyone's ideas valued? Did each group member fulfill the responsibilities of their role?
- Did the group finish the task in the time allotted?

## Activity Outline

Objective	Activity	Material;	Assessment	Homework
<ul style="list-style-type: none"> <li>• To conclude the postulate related to a line 1a.</li> <li>• To explain the reasoning behind the Postulate 1b, related to a plane.</li> <li>• To conclude Postulate 2.</li> </ul>	<ol style="list-style-type: none"> <li>1. The teacher asks the students several questions which leads to the Postulate 1a. These are:               <ul style="list-style-type: none"> <li>• Imagine that I draw a line on the board. Who can find its position? Students will draw different lines in different positions.</li> <li>• The teacher places a point on the board and asks the students to draw the required line which passes through the point. Students will draw different lines in different directions. Discuss previous homework.</li> <li>• The teacher adds the other point on the board and asks the students to draw a line which passes through two points. All the students will draw the same straight line which passes through two points. After a Socratic discussion led by the teacher, the students could conclude Postulate 1a. The teacher writes this postulate on the board with a drawing. <b>(Socratic Questioning)</b>. [VIDEO]</li> </ul> </li> <li>2.               <ol style="list-style-type: none"> <li>a) Inside the classroom the teacher determines any vertex and asks the students to show a plane that passes through the vertex. Students will show many planes.</li> <li>b) The teacher determines another vertex and asks the students to show a plane that passes through the two vertices. Students will show many planes.</li> <li>c) The teacher determines the third vertex and asks the students to show a plane passing through those three vertices . [VIDEO] The students will show only one plane. <b>(Socratic Questioning)</b></li> </ol> </li> <li>3. Students work in groups. The teacher writes questions on the board that lead to Postulate 2. These are:               <ul style="list-style-type: none"> <li>• Determine the straight lines of the book.</li> <li>• Determine the planes of the book.</li> <li>• Choose the line that many planes pass through. Finally they can conclude Postulate 2. Then the teacher writes this postulate on the board with drawings to illustrate the ideas. <b>(Cooperative Learning, Socratic Questioning)</b>.</li> </ul> </li> </ol>		EX 4, page 53 in the text. Exercise 1 and 7, page 59	Exercise 5, page 59

## Week 1 Day 3 Topic: Theorem 1

### Cooperative Learning – Group

In this lesson, groups are asked to report the results of their discussion with the class. Some teachers have the recorder give the report for the group. This approach often results in the rest of the group not paying attention because they know they don't have to report to the class. A better way to choose a reporter is to select one right before the group has to present their results to the class.

You can choose a random birthday and select the person in the group whose birthday is closest to that date to report. Another way to choose a reporter is to pick one role in the group right before the group is to present. For example, you could say a few minutes before the end of a group work time, "The quizzers will be reporting today." Not knowing who in the group is responsible for reporting keeps the whole group engaged.

### Modeling

Modeling is using physical objects like the paper and pen to represent concepts (plane and line). Using physical objects to model geometric ideas is important for students who don't draw well in three dimensions. Students can easily change the relationships between objects without making multiple drawings. Students illustrate an idea using something physical. Remember to discuss the weaknesses of this model. Pens have three dimensions, lines only have two dimensions. A line would intersect a plane in only one point, where a pen may intersect in a small circle.

### Cooperative Learning – Pairs

Pairs create maximum face-to-face interaction. No one gets left out in a group of two. Roles can be combined for groups of two. For example, one student can be quizzer/leader while the other student is recorder/materials manager. You should still give students a bit of time to talk about how well they worked together as a pair. If students are having difficulty getting along, help them resolve their conflicts with an emphasis on how to manage their own conflict next time.

## Activity Outline

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>• To conclude the number of planes that pass through a point and a straight line.</li> <li>• To prove Theorem 1.</li> <li>• To solve problems related to the theorem.</li> </ul>	<ol style="list-style-type: none"> <li>1. The teacher writes the following question on the board and distributes pens for each group and small cardboard shapes (square, semicircle, circle and triangle): The students try to answer, “How many planes pass through a straight line and a point outside it?” [VIDEO] Students brainstorm to answer the question using given materials. Then the teacher:               <ul style="list-style-type: none"> <li>• Asks for two groups to illustrate their work.</li> <li>• Writes the theorem on the board and mentions that any shape of a plane can be used in this theorem. <b>(Cooperative Learning - Group)</b></li> </ul> </li> <li>2. The teacher explains the theorem, what information is given and what information students need to provide in the proof. Students are then asked to prove the theorem as groups. The teacher then asks one student to illustrate his proof on board and the class is asked for other ideas. If the students do not succeed in proving the theorem, the teacher demonstrates how to prove it. <b>(Cooperative Learning - Group, Modeling)</b></li> <li>3. Assessment: The teacher solves Exercise 6, page 55</li> <li>4. The teacher writes a real-life exercise on the board (exercise) and shows the related drawing to the students using a projector. Students in pairs are challenged to solve the problem. [VIDEO] <b>(Cooperative Learning, Pairs)</b></li> </ol>	<ul style="list-style-type: none"> <li>• Pens</li> <li>• Small cardboard shapes (square, semicircle, circle and triangle)</li> <li>• Projector</li> </ul>	Exercise 6, page 55	

## Week 1 Day 4 Topic: Theorem 2

### Cooperative Learning - Brainstorming

In groups, students try to think of as many examples of two lines that intersect as a plane(s). Students can use lines and planes in the classroom to represent their ideas. They could also use other common classroom objects such as books and desks to model lines and planes. Brainstorming in a Cooperative Learning group should be done without judging the ideas each student contributes. Recorders should write down all the ideas.

### Cooperative Learning - Roles

Make sure to switch roles among members of Cooperative Learning groups to create individual accountability. They can stay in the same group as Day 3, but each student should take on a different role. Use the chart provided Appendix A for assigning roles. Once students get used to the chart, assigning roles will go quickly. Students share materials which creates positive interdependence. Give groups time to talk about how well their group worked together to teach group dynamics. Students sitting in small groups will ensure face-to-face interaction.

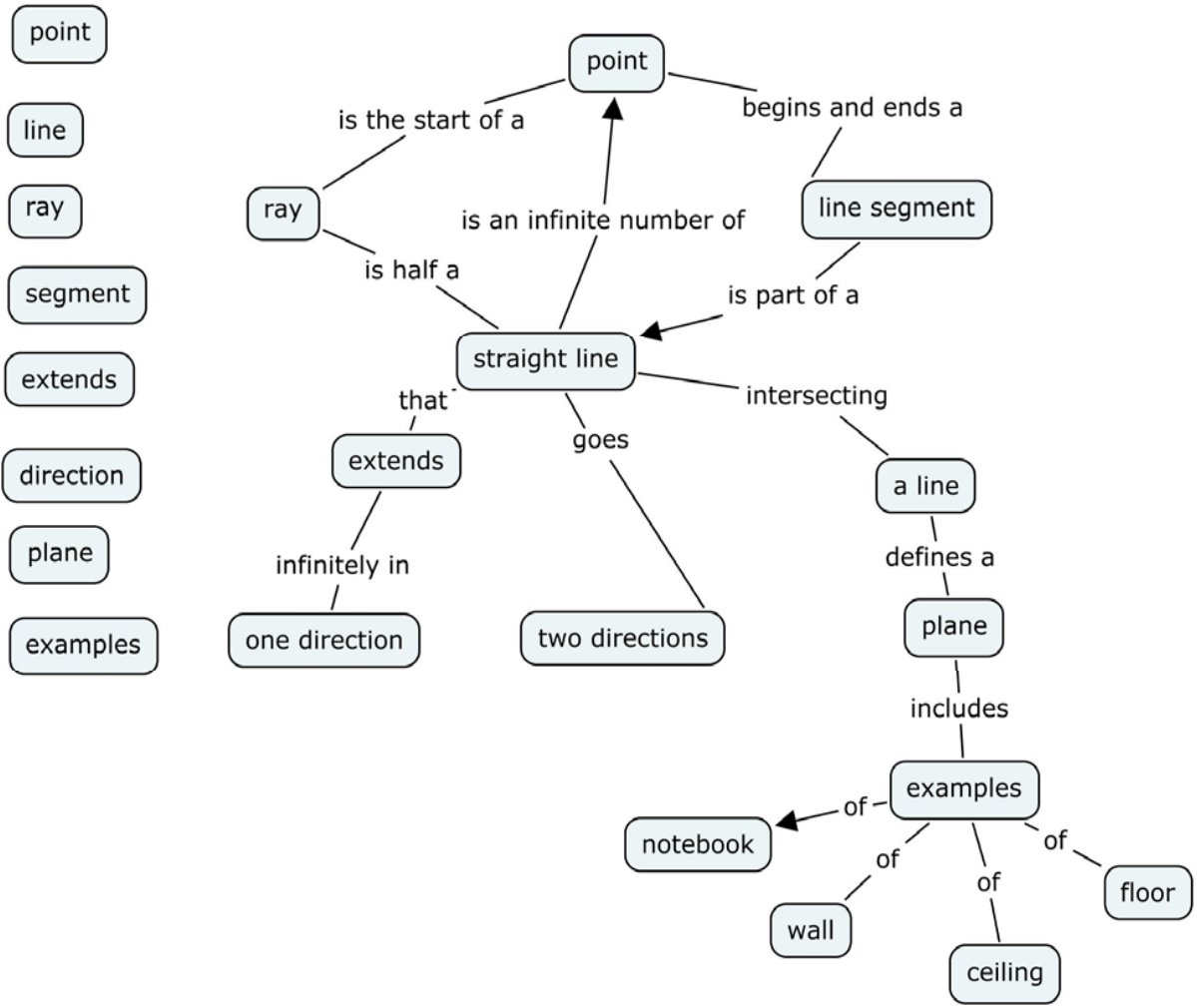
### Modeling

In addition to classroom objects, kabob sticks and paper are an important part of the lesson. They help students represent lines and a plane without having to draw them. It also lets students create many relationships between lines and planes quickly. The modeling materials are very useful for students who have trouble visualizing three-dimensional relationships from two-dimensional drawings. These students are often the most frustrated by geometry.

### Concept Mapping

Later in the lesson, students will be given an opportunity to add these terms to the concept map from Day 1.

Their map might look like this:



Notice new terms have been added to the “parking lot” on the left hand side of the map, including the examples created from brainstorming. See the online tutorial on Concept Mapping if you need to review how to construct a concept map.

## Activity Outline

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>• To conclude Theorem 2.</li> <li>• To prove Theorem 2.</li> </ul>	<ol style="list-style-type: none"> <li>1. The teacher writes the following question on the board and distributes two kabob sticks and papers to each group. How many planes can be determined by two intersected straight lines? [VIDEO] Students brainstorm answers to the question using given materials and examples from life. Then the teacher:               <ul style="list-style-type: none"> <li>• Asks two groups to illustrate their work.</li> <li>• Writes the theorem on the board and shows the model related to it. <b>(Cooperative Learning - Group Discussion and Brainstorming, Modeling)</b></li> </ul>               Students can think of other models besides the two kabob sticks. The teacher can show the Omani traditional fan to give students ideas.             </li> <li>2. The teacher analyzes the theorem and presents it as a request to prove the theorem. Next, the teacher asks one student to illustrate this proof on the board and asks the class for other ideas. Then the teacher gives a mini-lecture to prove the theorem if the students don't quite succeed in the proof. <b>(Mini-lecture, Cooperative Learning - Group)</b></li> <li>3. Assessment : Assign two exercises, from Appendix C (<b>one question work as individuals and other question work as cooperative pairs</b>)</li> <li>4. Students work in small groups to make concept maps with the rest of the brainstorming words on the board. Monitor students as they make their concept maps. They will post their maps on the board so the teacher can evaluate their work. [VIDEO] <b>(Cooperative Learning- Brain storming, Concept Maps)</b></li> </ol>	<ul style="list-style-type: none"> <li>• Kabob sticks</li> <li>• Papers</li> <li>• Omani traditional fan</li> <li>• Exercise sheets or overhead with problem(s) on it</li> </ul>	Exercise 7, page 56, Exercise 9, page 60	



## Week 1 Day 5 Topic: Theorem 3

### Cooperative Learning – Group Dynamics Rubric

Groups have been working together all week. It is time for them to make a more formal assessment of how well their group has been working together. Have each student in a group of four fill out this rubric on how well their group performed. These rubrics can be collected and used to assign participation points for the week. Judgment of participation is up to the teacher. Rubrics filled out by group members can provide the teacher with additional information.

<b>Group Work</b>	<b>Excellent = 3</b>	<b>Fair = 2</b>	<b>Need Improvement = 1</b>
How well did group work together?	Students worked well together and listened to their peers as they spoke.	Some students took the lead while others remained quiet.	Several students did not contribute to the discussion.
Did each person fulfill the responsibilities of their group role?	All students were able to contribute. Each idea was considered in the consensus for the final answers.	One leader controlled most decisions. Some students did not contribute to the final answers.	Students argued and never came to agreement on one answer for each problem.
Did the group value other groups' explanations?	All students in the group listened attentively while other groups explained their answers.	Most students paid attention to other groups when they were explaining their answers.	Little attention was given to another group that was presenting.
Are the answers recorded clearly and accurately?	The answers are well written, correctly sequenced and include all necessary steps and information	Answers are not clearly written, the sequence of steps isn't always correct and some information is missing.	Answers were poorly written, partially sequenced with many key steps missing.

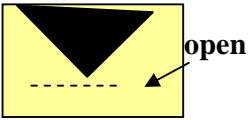
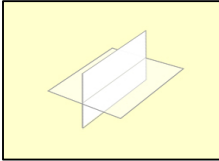
### Modeling

Students use the pieces of cork to represent two planes in different shapes. A slit is cut in one plane so the planes can intersect as a line. The result of the intersection of two planes is a line. In this model the slit is the line of intersection.

## Preparing Review Problems

When choosing questions for the review, choose some easy questions and some more difficult questions. Different types of questions are good for encouraging students to think in new ways. It also helps them prepare for exams. If possible, assign different problems to different Cooperative Learning groups. Having students look at their errors helps them to understand why they made them and how to avoid making the same mistake again.

## Activity Outline

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>To prove Theorem 3.</li> <li>To apply Theorems 1, 2 and 3 to practical problems.</li> </ul>	<p>1. The teacher writes Theorem 3 on board and distributes two colored planes to each group (pieces of cork as follows)</p> <div style="text-align: center;">   </div> <p>The teacher asks students to demonstrate the theorem using the given materials. Students brainstorm in cooperative groups to illustrate the theorem with the materials they have been given and with the guidance of the teacher. Then the teacher:</p> <ul style="list-style-type: none"> <li>Selects two groups to illustrate their work. <b>(Modeling, Cooperative Learning - Groups)</b> [VIDEO]</li> <li>Presents a mini-lecture and discusses with students how to prove Theorem 3. <b>(Discussion)</b></li> </ul> <p>2. Assessment : Assign Exercise 3, page 59.</p> <p>3. Paired students work to solve worksheet problems (Appendix C) involving different ways to review Theorems 1, 2 and 3. (Cooperative Learning - Groups)</p>	Two colored planes (pieces of cork)	Exercise 3 and 4, page 59	Complete the assigned problems from Appendix C.

## Week 2 Day 1 Topic: Postulate Related to a Space

### Modeling

In this activity, the edges of a shape or line segments are represented by kabob sticks. The cork can be used as a base for the models. Students can place the kabob sticks in the cork to make three-dimensional shapes. It is impossible to make a three-dimensional model with less than 6 kabob sticks (triangular pyramid). Students should conclude that it takes at least 4 points to define space. Remind students to include the edges and vertices that are in the plane of the cork.

If you have enough materials, provide cork and kabob sticks for each student so that each student will get to spend the maximum time manipulating the modeling materials. If you do not have enough materials for each student, remind the materials manager to make sure each student gets a chance to build a three-dimensional model.

### Cooperative Groups

You can choose to change the groups of students who sit together. This provides more opportunities for students to get to know other students in the class. Making new groups of four also gives group members who are getting frustrated with their group a fresh start. Use the role chart in the Appendix to assign roles to new groups of students.

### Activity Outline

Objective	Activity	Materials	Assessment	Homework
To conclude Postulate 3.	<ol style="list-style-type: none"> <li>The teacher distributes kabob sticks and pieces of cork to each group, and asks the students to make three-dimensional models which consist of 3, 4, 5 or more vertices within 15 minutes. Then the teacher selects some groups to demonstrate their work. No group can make a three-dimensional model with less than 4 vertices. The teacher should help the class connect their modeling activity with the number of points (4) needed to define space. [VIDEO] (<b>Cooperative Learning – Group, Modeling</b>)</li> <li>The teacher writes the Postulate 3 on the board and defines the geometric meaning of “space.” (<b>Mini-lecture</b>)</li> <li>Assessment: Assign Exercise 2, page 62.</li> </ol>	<ul style="list-style-type: none"> <li>Kabob sticks</li> <li>Pieces of cork</li> </ul>	Exercise 2, page 62	Exercise 5, page 70

## Week 2 Day 2 Topic: Lines and Planes in Space

### Modeling

In this activity kabob sticks represent lines, and the cork represents planes. Two kabob sticks can be made parallel, intersecting or skewed (not parallel and not intersecting).

### Cooperative Learning

Groups of four can be split into two groups of two for pair activities or students who are already sitting together at the same desk can work together. One of the pair can be the artist, the other student can write down the pair's ideas in words. Give the pairs a minute or two at the end of the activity to discuss how well they worked together.

### Activity Outline

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>• To conclude the different relationships between two straight lines.</li> <li>• To conclude the different relationship between a line and a plane.</li> </ul>	<ol style="list-style-type: none"> <li>1. The teacher distributes kabob sticks and pieces of cork for each pair of students and asks them the following question: What are the different positions of the two kabob sticks? Describe the relationships between them and illustrate them with a drawing. [VIDEO] <b>(Cooperative Learning, pairs, Modeling)</b> The teacher asks for student ideas and writes them on the board with the accompanying drawings.</li> <li>2. The teacher uses the example on page 64 and gives examples from the class to illustrate the relationships between two lines. <b>(Mini-lecture)</b> Assessment: Exercise 3, page 64.</li> <li>3. The teacher asks pairs of students to choose any plane and line in the classroom to conclude relationships between them. The teacher then selects some students to illustrate these relationships. <b>(Cooperative Learning - Pairs)</b></li> <li>4. The teacher shows the results using a projector. <b>(Mini-lecture)</b></li> <li>5. Assessment: Assign exercises Exercise 4, page 65 <b>(Cooperative Learning - Groups)</b></li> </ol>	<ul style="list-style-type: none"> <li>• Kabob sticks</li> <li>• Pieces of cork</li> </ul>	Exercise 3, page 64	The teacher will choose from the worksheet.

## Week 2 Day 3 Topic: Theorem 4

### Modeling the Theorem

Students can use a corner of the classroom to represent two intersecting lines (the two edges of the floor) and a line perpendicular to both of the intersecting lines (the corner of the wall). This activity can be done quickly. It gives students a chance to represent their thinking with three-dimensional objects.

### Modeling the Result of the Theorem

Students use their notebooks and pens to model two parallel lines that are perpendicular to the plane. Students can also model using the corners between the walls and floor of the classroom.

### Journaling

In their journals, students write their predictions about the relationship between two parallel lines in which one is perpendicular to a plane. Students can make a drawing if they would like. The purpose of this exercise is to get students to think about what the theorem means.

### Inquiry

This activity uses Inquiry since the students make predictions before they learn the result of the theorem. They model to test their prediction. Don't tell the students the answer before they have had some time to think about it.

## Activity Outline

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>• To explain Theorem 4, page 67.</li> <li>• To prove the theorem.</li> <li>• To explain the result related to the theorem.</li> </ul>	<ol style="list-style-type: none"> <li>1. The teacher writes the theorem on the board and asks the students to explain the theorem using the physical classroom to model the intersection of lines and planes. [VIDEO] <b>(Modeling, Cooperative Learning - Pairs).</b></li> <li>2. The teacher gives a mini-lecture and discusses with students how to prove the theorem. <b>(Mini-lecture)</b></li> <li>3. The teacher asks students individually to answer the following question using their note books and pens to illustrate their answers as they write in their journal: If there are two parallel lines and one of them is perpendicular to a plane, is the other line perpendicular to the same plane? <i>Answer: Yes, both the parallel lines will be perpendicular to the plane. (Journaling)</i></li> </ol> <div data-bbox="726 875 1005 1109" style="text-align: center;"> </div> <ol style="list-style-type: none"> <li>4. The teacher selects some students to show their ideas. Then the teacher gives a mini-lecture describing the relationship between a plane and a line using a drawing to illustrate the theorem. <b>(Inquiry, Modeling, Individual Activity and Mini-lecture )</b></li> <li>5. The teacher assigns problems from Appendix C as an individual activity.</li> </ol>		<p>The teacher chooses problems from Appendix C.</p>	<p>The teacher will choose a homework assignment from Appendix C.</p>

## Week 2 Day 4 Topic: Relationship between Two Planes in Space

### Modeling

Students use cardboard to represent planes. They will find that planes can intersect a line or can be parallel. It is important to remind students that the cardboard would extend infinitely in any direction. Students should conclude that two planes intersect in a straight line.

### Cooperative Learning

Don't forget to rotate roles within each group using the chart in Appendix A. As students work to find the relationship between two planes, remind the student in the recorder role to draw different representations of the group's modeling. When students work as quads, you can split the groups of four into groups of two. When students work in pairs, make sure both students are contributing. If some students are not participating, remind them that they may be selected to present their answers to the class.

### Activity Outline

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"><li>To conclude the relationship between two planes.</li><li>To solve problems about lines and planes in space.</li></ul>	<ol style="list-style-type: none"><li>The teacher distributes two colored planes to each group and asks the students to do the activity on page 68, in groups, and answer the questions. <b>(Cooperative Learning - Groups, Modeling)</b>.</li><li>The teacher asks each group to show their model and present their conclusions.</li><li>Assessment: The teacher assigns Exercise 6, page 70 and Exercise 8 as a pairs activity.</li><li>Students work in pairs to solve the review problems. <b>(Cooperative Learning - Pairs)</b>.</li></ol>	Two colored planes (hard paper)	Exercise 6, page 70 and review problems from Appendix C.	Complete the problems from Appendix C.

## Week 2 Day 5 Topic: Coordinates of a Point in Three-Dimensional Space

### Modeling

The teacher assigns each group a different point in the room to measure. The students must decide how to describe this point with measurements relative to the walls and floor or ceiling of the classroom. The teacher will explain how to write their measurements in a bracket in the  $x, y, z$  form. If students stand on desks or chairs to measure something close to the ceiling, monitor them to make sure they are safe. Help students decide if it makes more sense to measure the distances in centimeters or meters.

### Cooperative Groups

Cooperative groups should spend some time writing about how well their group worked together. They can use the following rubric as a guide or simply circle the choice they think is most appropriate for their group. Each student should evaluate their group.

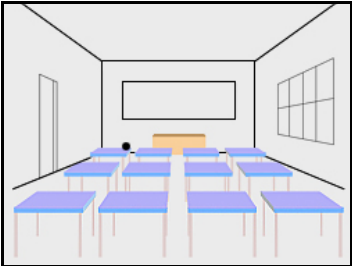
<b>Group Work</b>	<b>Excellent = 3</b>	<b>Fair = 2</b>	<b>Needs Improvement = 1</b>
How well did the group work together?	Students worked well together and listened to their peers as they spoke.	Some students took the lead while others remained quiet.	Several students did not contribute to the discussion.
Did each person fulfill the responsibilities of their group role?	All students were able to contribute. Each idea was considered in the consensus for the final answers.	One leader controlled most decisions. Some students did not contribute to the final answers.	Students argued and never came to agreement on one answer for each problem.
Did the group value other groups' explanations?	All students in the group listened attentively while other groups explained their answers.	Most students paid attention to other groups when they were explaining their answers.	Little attention was given to another group that was presenting.
Are the answers recorded clearly and accurately?	The answers are well written, correctly sequenced and include all necessary steps and information.	Answers are somewhat unclearly written, the sequence of steps isn't always correct and some information is missing.	Answers were poorly written, partially sequenced with many key steps missing.



## Game

In groups of four, students play the game on page 71 of the math text ( Activity1). Student one thinks of a point. Student two asks questions about the point. Students three and four try to guess the point from the answers given to student two's questions. Each student takes a turn thinking of a point. The teacher can demonstrate the game before the students play. Roles for this activity are fairly easy to assign. Students just rotate thinking of points, asking questions, and guessing. Students can each write the coordinates of the point using the  $(x, y, z)$  system. They should also write what they are using as the origin in the classroom and what the points represent. These sheets can be collected as part of the assessment.

Students use the  $x, y, z$  model from The Teacher Materials Department to demonstrate locating the point on page 73.

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>• To recognize the coordinate system.</li> <li>• To describe a point in three-dimensional space using the <math>x, y, z</math> coordinate system.</li> </ul>	<p>1. The teacher assigns each group a point in the room. Two groups should have the same point so they can check their measurements with each other. [VIDEO] <b>(Modeling, Cooperative Learning - Groups)</b>  <i>Examples: a hanging light, the corner of a desk, or some other classroom landmark.</i></p>  <p>The teacher identifies the strength and weakness of each explanation.</p> <p>2. The teacher explains:</p> <ol style="list-style-type: none"> <li>a) How the <math>x, y, z</math> coordinate system can be used to describe points in space.</li> <li>b) How to write points in brackets as <math>(x,y,z)</math>. Then the teacher discusses the example on page 73 with the class. <b>(Mini-lecture)</b></li> </ol>	<ul style="list-style-type: none"> <li>• Ruler</li> <li>• Metric tape</li> <li>• Model of <math>x, y, z</math> coordinate system.</li> </ul>	<p>Explain how to measure the point in three-dimensional space using the <math>x, y, z</math> coordinate system.</p> <p>Exercise 3 and 4, page 73</p>	<p>Give an example from Exercise 1a, page 80.</p>

Objective	Activity	Materials	Assessment	Homework
	<p>3. Students do Activity 1, page 71. In this activity, students brainstorm how to describe and measure the point. Then they measure where the point is in relation to where the classroom is located. Help the students realize they should measure perpendicular to each wall and the floor or ceiling. [VIDEO] <b>(Cooperative Learning - Group, Modeling)</b></p> <p>4. Assessment: Assign Exercises 3 and 4, page 73</p>			

## Week 3 Day 1 Topic: Coordinates of a Point in Three-Dimensional Space

### Modeling

The teacher uses an  $x, y, z$  model to demonstrate how to place a point in space. Students practice using the  $x, y, z$  models by locating the point in the example on pages 72-73 in the text. Students should also try to draw this point in their notebooks. If students finish the example they can start working on problems on pages 72-73

Objective	Activity	Materials	Assessment	Homework
<p>To draw a given <math>(x, y, z)</math> point in three-dimensional space.</p>	<ol style="list-style-type: none"> <li>1. The teacher uses the <math>x, y, z</math> coordinate system as a model to show students how to use coordinates to place a point in three-dimensional space.</li> <li>2. The teacher shows students how to draw a point in a three-dimensional space using the board. <b>(Mini-lecture)</b></li> <li>3. Have students practice drawing a point. They can use their three-dimensional models to help them draw. [VIDEO] Check the students' drawings to make sure they are correct. Ask a member of the class with the correct answer to explain how they drew their point. Assign exercises 2-5, pages 72-73 in the text. Check student work. <b>(Individual Activity, Modeling, if they use the <math>x, y, z</math> model)</b></li> <li>4. Assign problems from Appendix C to complete in class if there is time; or give this problem as homework if there are enough copies for each student.</li> </ol>	<ul style="list-style-type: none"> <li>• Model of <math>x, y, z</math> coordinate system.</li> <li>• Copies of the review problems for each group or for each student if it will be assigned as homework.</li> </ul>	<p>Individuals practice drawing a point given the coordinates, following the steps on the page 73 in the text.</p> <p>Exercises 2-5, pages 72-73</p>	<p>Review problems from Appendix C.</p>

## Week 3 Day 2 Topic: Distance between Two Points in Three-Dimensional Space

### Cooperative Groups

New groups of students can be arranged for the third week of the unit. It is also appropriate for the same group of four students to remain together for the entire unit. How long groups stay together is up to the teacher. When students are just starting to use Cooperative Learning, switching groups more often may help ease some group conflict if needed. As students get better at resolving their own conflicts, they may profit from staying in the same groups longer.

### Modeling

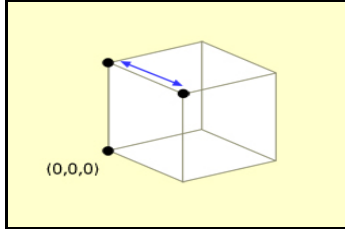
The students use boxes in class to measure two points in the same plane. Teachers should bring in several different kinds of boxes (tissue, cereal, shoe, biscuits, crackers, etc.) and mark three points in the same plane. One point should be a different color to represent the origin.

As a homework assignment, students build boxes from papers given out by the teacher with an outline of the box on the paper. Blackline Masters for these boxes can be found in Appendix D. At home the students will cut out the box and glue or tape it together. The points they measure will be marked on the paper pattern, when they glue the box the points will appear. The origin will be indicated on the box as well. The homework assignment will be more challenging as it requires students to find the distance between points that are not in the same plane.

### Problem-based Learning

In this strategy students have to extend their thinking from two dimensions to three. Instead of just giving them the formula, they must use what they learned about two dimensions to write a new formula for three dimensions. They use a model to help them write the new formula. The formula for three dimensions they use to solve the problem given in class will be similar to the formula for two dimensions since the points are in the same plane.

## Activity Outline

Objective	Activity	Materials	Assessment	Homework
<p>To find the distance between two points in three-dimensional space.</p>	<ol style="list-style-type: none"> <li data-bbox="491 365 1222 430">1. The teacher gives coordinates of two points in two-dimensional space.           <div data-bbox="684 464 1026 691" style="text-align: center;">  </div> </li> <li data-bbox="491 732 1222 829">2. In pairs, students calculate the distance between the two points using the distance formula for two dimensions. <b>(Cooperative Learning - Pairs)</b></li> <li data-bbox="491 837 1222 1000">3. The teacher asks students how they would calculate the distance between two points in three-dimensional space. Students brainstorm ideas on how they will calculate the distance above. [VIDEO] <b>(Brainstorm, Problem-based Learning)</b></li> <li data-bbox="491 1008 1222 1269">4. The teacher passes out boxes of different sizes and shapes. Students are challenged to find the distance between two opposite corners (on the same plane). They do the activity on page 74 of the text as groups. Students should be able measure the distance with a ruler and then use the formula. Students can use the same strategy for their homework assignment. <b>(Cooperative Learning - Groups, Modeling)</b></li> </ol>	<ul style="list-style-type: none"> <li data-bbox="1247 365 1436 500">• Papers with box outlines on them (see Appendix D)</li> <li data-bbox="1247 508 1346 532">• Ruler</li> </ul>	<p>Exercise 6, page 74</p>	<p>Students will be assigned different boxes to cut out and assemble, and then they will find the distance between two points on the box that are not in the same plane. They will check with students who had the same box in the next class to verify their answer.</p>

## Week 3 Day 3 Topic: Midpoint in Three-Dimensional Space

### Pre-Class Preparation

Before class begins, put a small piece of clay on each model to hold the kabob stick. In order to prevent students from copying each other, give each group different endpoints for their kabob stick. Use different lengths of kabob stick for each group as well.

Before class begins, make transparencies of both the photo of the moon and sun as well as the transparency outlining how they are projected on to the plane of the sea. You can find both transparencies in Appendix B.

### Modeling

First, the student finds the coordinates of the two endpoints of the kabob stick in the model. Then the student uses the ruler to measure the distance between the two endpoints and measures the midpoint as well. After that, the student uses the formula for three-dimensional space to find the midpoint. Students may not have exactly the same values for the calculated and measured mid-point because their measurements may have some error in them.

### Cooperative Learning

Task 3 can be done in pairs if there are enough materials. When students are working in pairs on a worksheet, both of them should fill out a worksheet. Having all group members fill out their own worksheet helps create individual accountability in the pair. To create positive interdependence in the pair, either of the pair's worksheet can be turned in for a grade for both pairs.

## Activity Outline

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>To find the midpoint in three-dimensional space.</li> <li>To define the term “projection” as it is used in geometry.</li> </ul>	<ol style="list-style-type: none"> <li>Assign student pairs the text exercise to review that uses the midpoint formula in two-dimensional space. (<b>Cooperative Learning - Pairs</b>)</li> <li>The teacher distributes a model to each group. The model consists of sharp endpoints of kabob stick inside a box. The teacher gives different lengths of sticks to each group. [VIDEO] (<b>Modeling</b>)</li> <li>Students work in groups to:               <ul style="list-style-type: none"> <li>Use a ruler to measure the distance between one of the endpoints and the mark.</li> <li>Find the coordinates of the endpoints of the kabob stick from the model.</li> <li>Find the midpoint coordinates of the line using the formula in three-dimensional space.</li> </ul> </li> </ol> <div data-bbox="676 808 982 1065" data-label="Image"> </div> <ol style="list-style-type: none"> <li> <ul style="list-style-type: none"> <li>Mark the calculated midpoints on the stick in the model.</li> <li>Students compare the measured and the calculated midpoint distance. (<b>Cooperative Learning - Groups</b>)</li> </ul> </li> <li>Assessment: The teacher assigns Exercise 7, page 75. (<b>Cooperative Learning – Pairs</b>)</li> <li>On the board, the teacher draws a point above a line, a point above a plane and a line above the plane. Ask the students to draw what would it would look like if the points and line fell</li> </ol>	<ul style="list-style-type: none"> <li>Model of <math>x, y, z</math> coordinate system.</li> <li>The sharp endpoints of a kabob stuck inside a model</li> <li>Small piece of clay to put the stick in</li> </ul>	Exercise 7, page 75 Define the projection of a point on a line/a plane.	Prove the lemma of theorem on page 75.

Objective	Activity	Materials	Assessment	Homework
	<p>on the geometric structure below it. <b>(Discussion)</b></p> <ol style="list-style-type: none"> <li>6. Use the students' drawings to define projection.</li> <li>7. The teacher shows the students a photo of the moon and sun over the sea. After that the teacher adds another transparency on top of the photo that outlines how both the sun and the moon is projected on the sea [double transparency].</li> <li>8. Assessment.</li> </ol>			



## Week 3 Day 4 Topic: Perpendicular Projection

### Modeling in Cooperative Groups

Students should model the theorem and the lemma with the kabob sticks, cardboard and clay. If possible, more than one set of materials should be available to each group so all students can get a chance to handle the material as possible. Although, it is good for everyone to have their own materials, sharing promotes positive interdependence.

A reporter should be chosen from the group at random to encourage individual accountability. Any group member should be able to present to the class. As students become more accustomed to explaining their work to the class, they should be reminded of what makes a good presentation. Eye contact, use of appropriate language, and posture should be discussed with the class. These social skills are also useful in smaller groups.

### Activity Outline

Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>• To prove Theorem 4.</li> <li>• To recognize the converse of the theorem.</li> </ul>	<ol style="list-style-type: none"> <li>1. The teacher writes the statement of Theorem 4 on the board and asks the students (in groups) to represent the theorem by using materials (kabob sticks, clay and cardboard).</li> <li>2. The teacher:               <ul style="list-style-type: none"> <li>• Selects some groups to show their work and then clarifies the theorem. Then the teacher gives the students a chance to prove the theorem</li> <li>• Discuss the proof with the students. (<b>Cooperative Learning - Groups, Modeling</b>)</li> </ul> </li> <li>3. The teacher writes the statement of converse of the theorem on the board and asks the students (in groups) to demonstrate the converse of the theorem by using previous materials (kabob sticks, clay and hard paper). (<b>Cooperative Learning - Groups, Modeling</b>)</li> <li>4. Assessment: Example 5, page 77 in the text; Exercise5, page 82.</li> </ol>	<ul style="list-style-type: none"> <li>• Kabob sticks</li> <li>• Clay</li> <li>• Cardboard</li> </ul>	Exercise 5, page 82	

## Week 3 Day 5 Topic: Dihedral Angle

### Teacher Preparation

Bring some objects from home that show dihedral angles: cartons, CD cases, irregular shapes or interesting boxes (Example: Toblerone candy box).

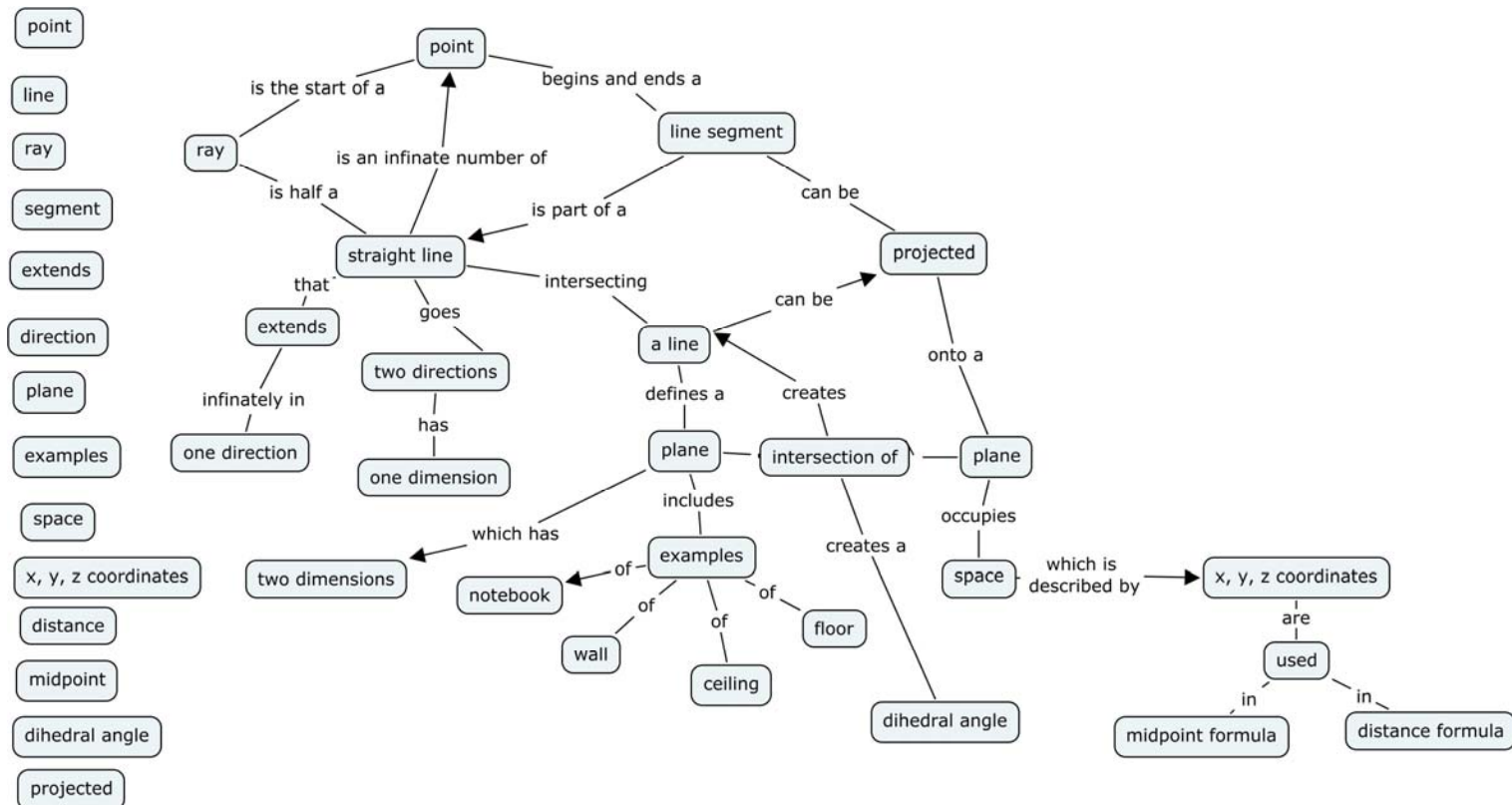
### Cooperative Groups

As the end of the last week approaches, students should sum up what they have learned about working in a group. In addition to filling out the rubric below students should write a few sentences on what makes a good group member and how they became a better group member during the unit.

<b>Group Work</b>	<b>Excellent = 3</b>	<b>Fair = 2</b>	<b>Need Improvement = 1</b>
How well did the group work together?	Students worked well together and listened to their peers as they spoke.	Some students took the lead while others remained quiet.	Several students did not contribute to the discussion.
Did each person fulfill the responsibilities of their group role?	All students were able to contribute. Each idea was considered in the consensus for the final answers.	One leader controlled most decisions. Some students did not contribute to the final answers.	Students argued and never came to agreement on one answer for each problem.
Did the group value other groups' explanations?	All students in the group listened attentively while other groups explained their answers.	Most students paid attention to other groups when they were explaining their answers.	Little attention was given to another group that was presenting.
Are the answers recorded clearly and accurately?	The answers are well written, correctly sequenced and include all necessary steps and information.	Answers are somewhat unclearly written, the sequence of steps isn't always correct and some information is missing.	Answers were poorly written, partially sequenced with many key steps missing.

## Cooperative Learning - Brainstorming

As groups suggest real-life examples of dihedral angles, the recorder should write down all their ideas without judging. If there is time, groups can add the real life examples of dihedral angles to their concept map along with other new terms learned since week 1. If desired, the concept map with all terms on it from the unit can be used as part of a final assessment. A final concept map might look like this:



Notice the new terms in the “parking lot” on the left. There are many ways to represent the relationships between these terms. This map is one of many that could accurately describe the terms and how they are related. See the online tutorial on Concept Mapping if you need to review how to construct a concept map.

## Activity Outline

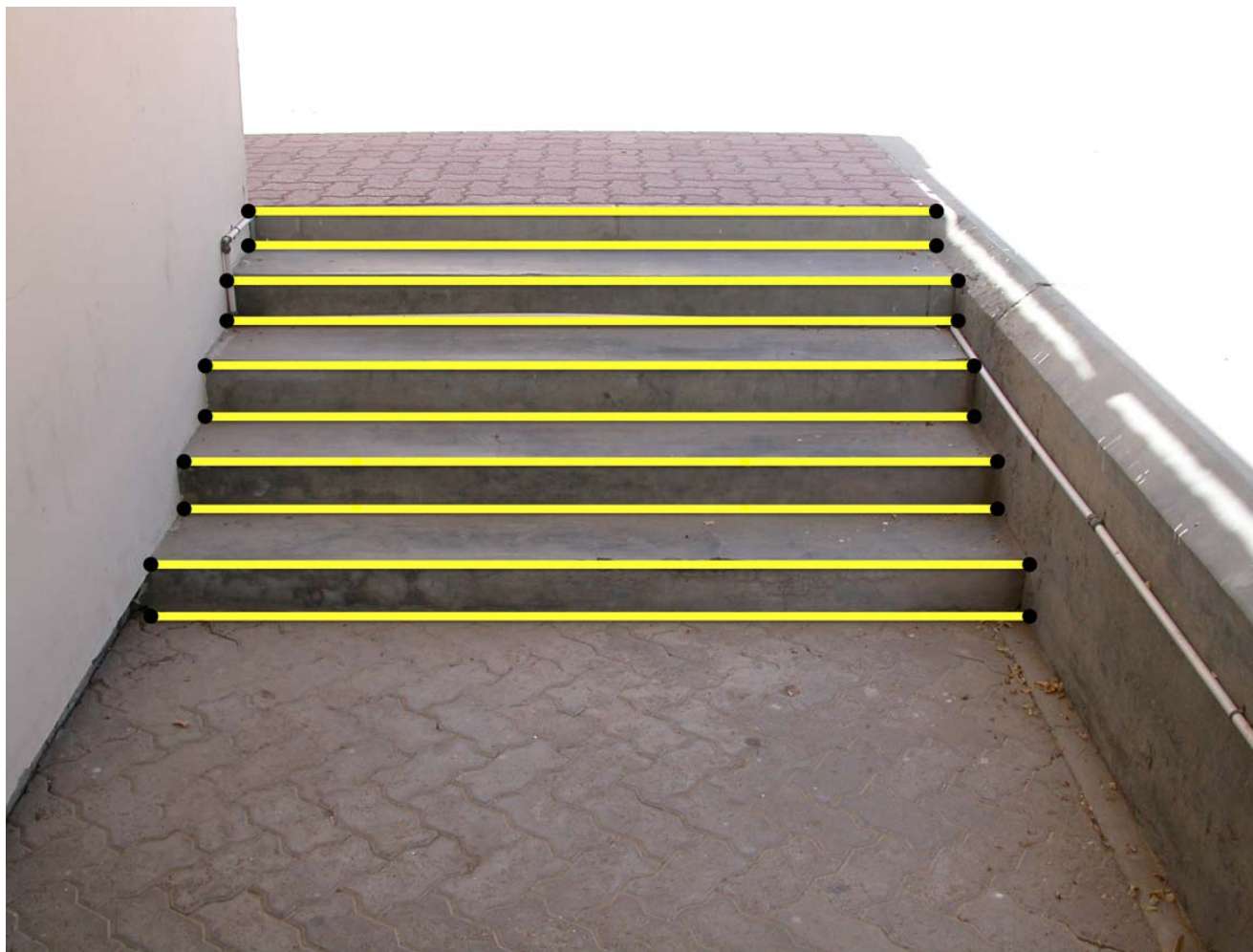
Objective	Activity	Materials	Assessment	Homework
<ul style="list-style-type: none"> <li>• To define a dihedral angle.</li> <li>• To measure a dihedral angle.</li> </ul>	<ol style="list-style-type: none"> <li>1. Discussion of homework. The teacher proves converse of the theorem if students have not been able to do so in their homework.</li> <li>2. The teacher gives the students the definition of a dihedral angle and asks the class for real life examples of two planes intersecting (door, book covers, CD case). [VIDEO] (<b>Cooperative Learning Groups brainstorm practical examples</b>).</li> <li>3. The teacher gives a mini-lecture on how to read and write dihedral angles on the board. The teacher draws examples of dihedral angles on the board and students work together to name them. (<b>Mini-lecture, Discussion</b>)</li> <li>4. The teacher distributes plastic, regular and irregular models of fixed and different angles to each group.</li> <li>5. Students in groups brainstorm a method of measuring dihedral angles.</li> <li>6. The teacher selects groups to present. The group with no regular models will have difficulties measuring dihedral angles. (<b>Cooperative Learning - Brainstorming Circle, Modeling</b>)</li> <li>7. The teacher gives a mini-lecture on the correct method of measuring a dihedral angle using the given materials. [VIDEO] (<b>Mini-lecture</b>)</li> <li>8. Assessment: The teacher assigns Exercise 12 a and b, page 79 as a pair activity.</li> </ol>	<ul style="list-style-type: none"> <li>• Plastic regular and irregular models of fixed and different angles</li> </ul>	<p>List real life examples of a dihedral angle.</p> <p>Exercise 12, a and b, page 79</p>	<ul style="list-style-type: none"> <li>• Define a dihedral angle.</li> <li>• Measure a dihedral angle.</li> </ul>

## Appendix A: Role Chart for Cooperative Learning

<b>Day</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
<b>A</b>	Leader	Materials Manager	Quizzer	Recorder
<b>B</b>	Recorder	Leader	Materials Manager	Quizzer
<b>C</b>	Quizzer	Recorder	Leader	Materials Manager
<b>D</b>	Materials Manager	Quizzer	Recorder	Leader

Today is a(n) A B C D day.

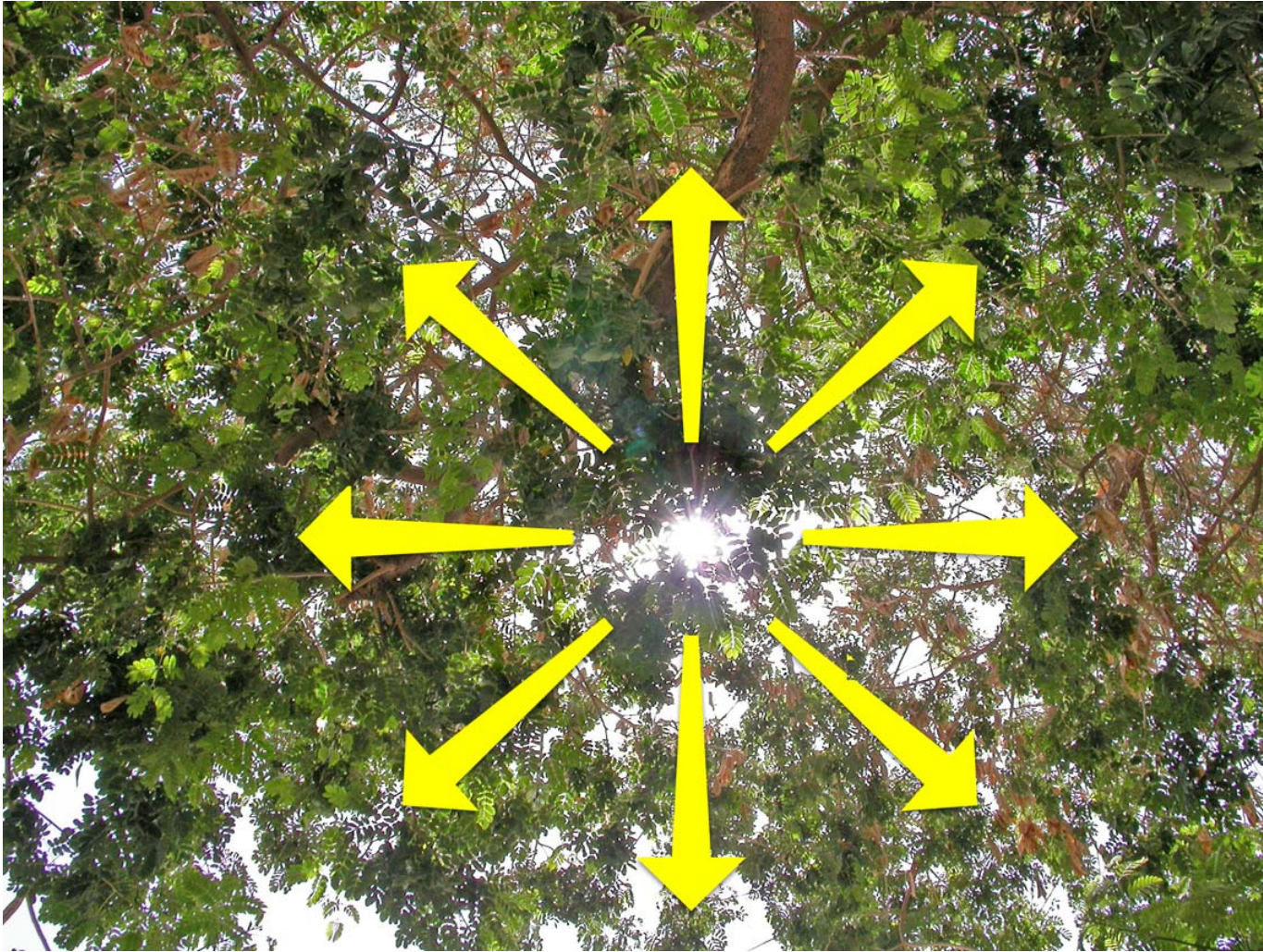
## Appendix B: Photos for Lesson 1



Line Segments

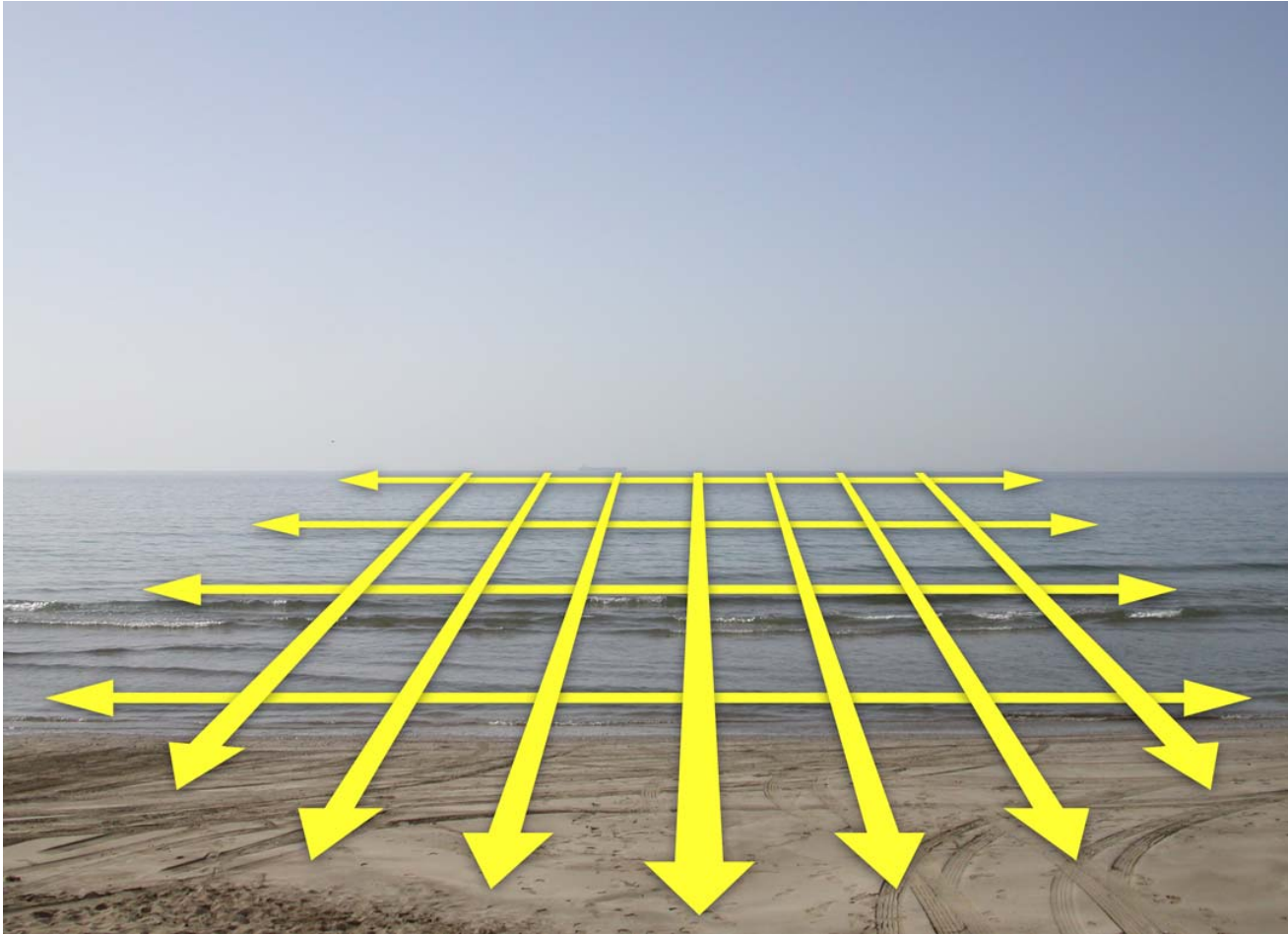


Rays (tree branches)



Point (Sun) Rays of Sunshine





Plane (Surface of the Sea)



Space (Dubai Airport)



Dihedral Angle (Intersection of Two Walls)

## Appendix C: Additional Problems

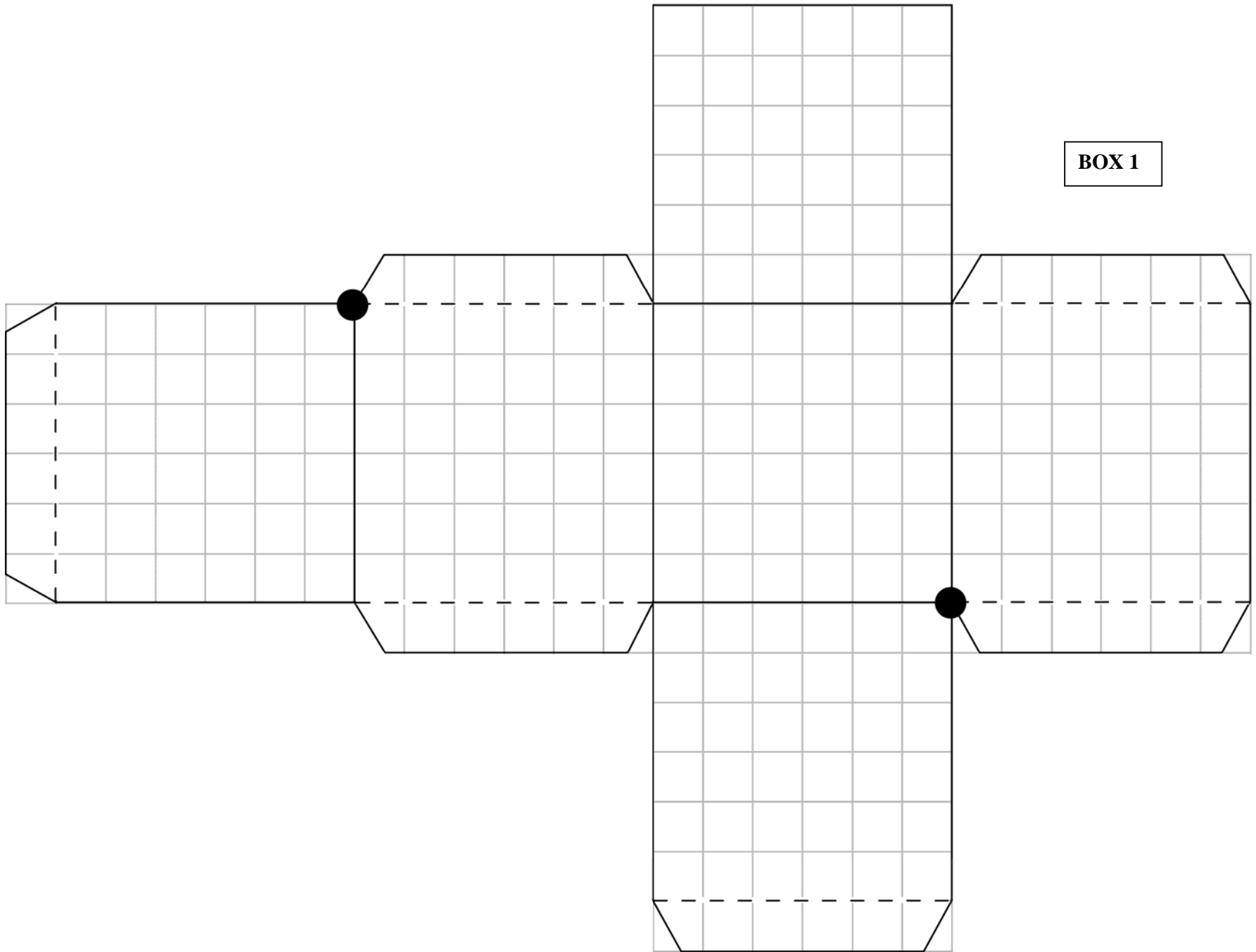
The additional problems can be found on page 76 of the Arabic version of this document. They were not translated into English.



## Appendix D: Box Diagrams for Week 3 Day 2

Have students cut out and glue the box diagrams on the following pages as part of their homework for Week 3, Day 2.

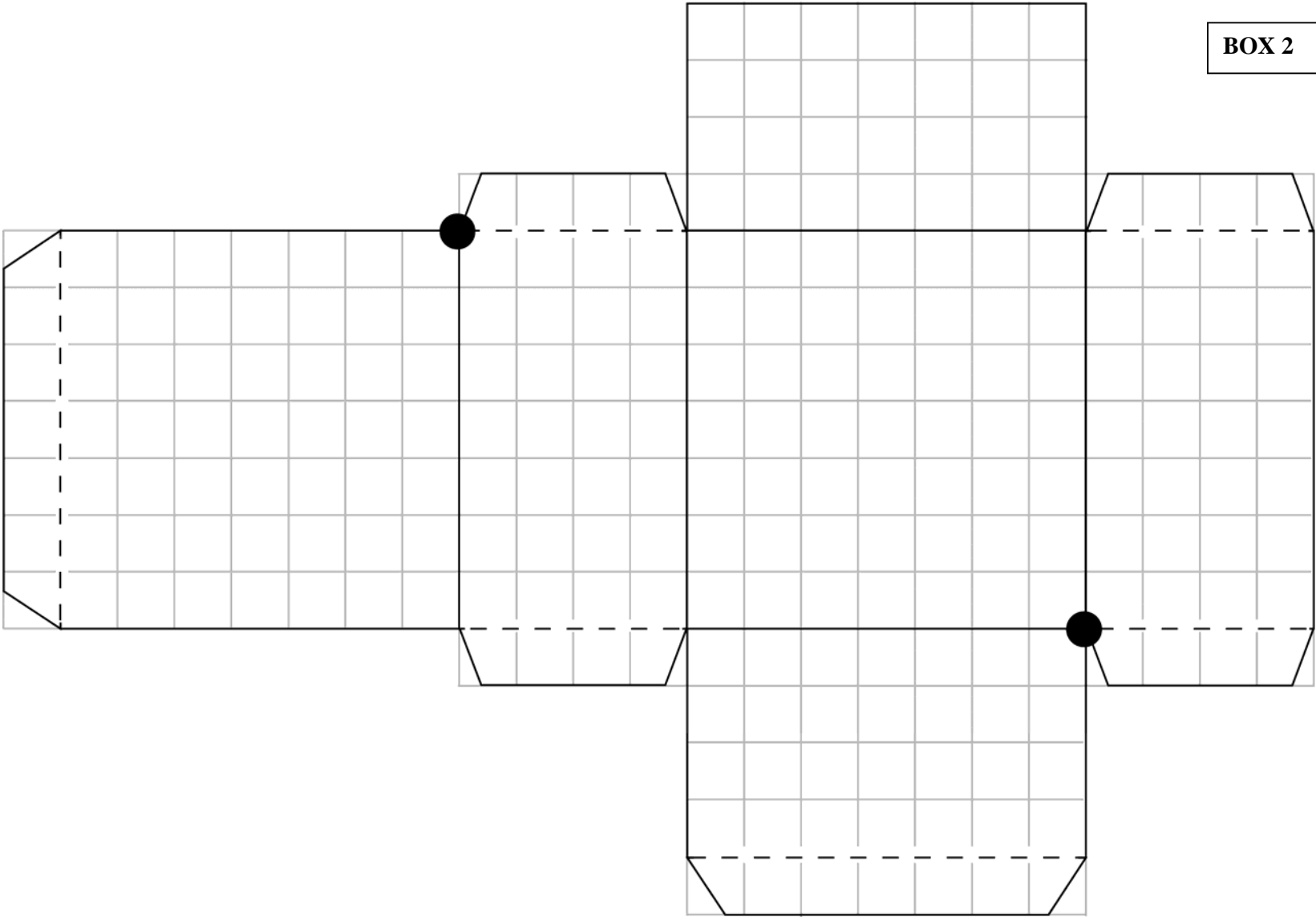
**BOX 1**





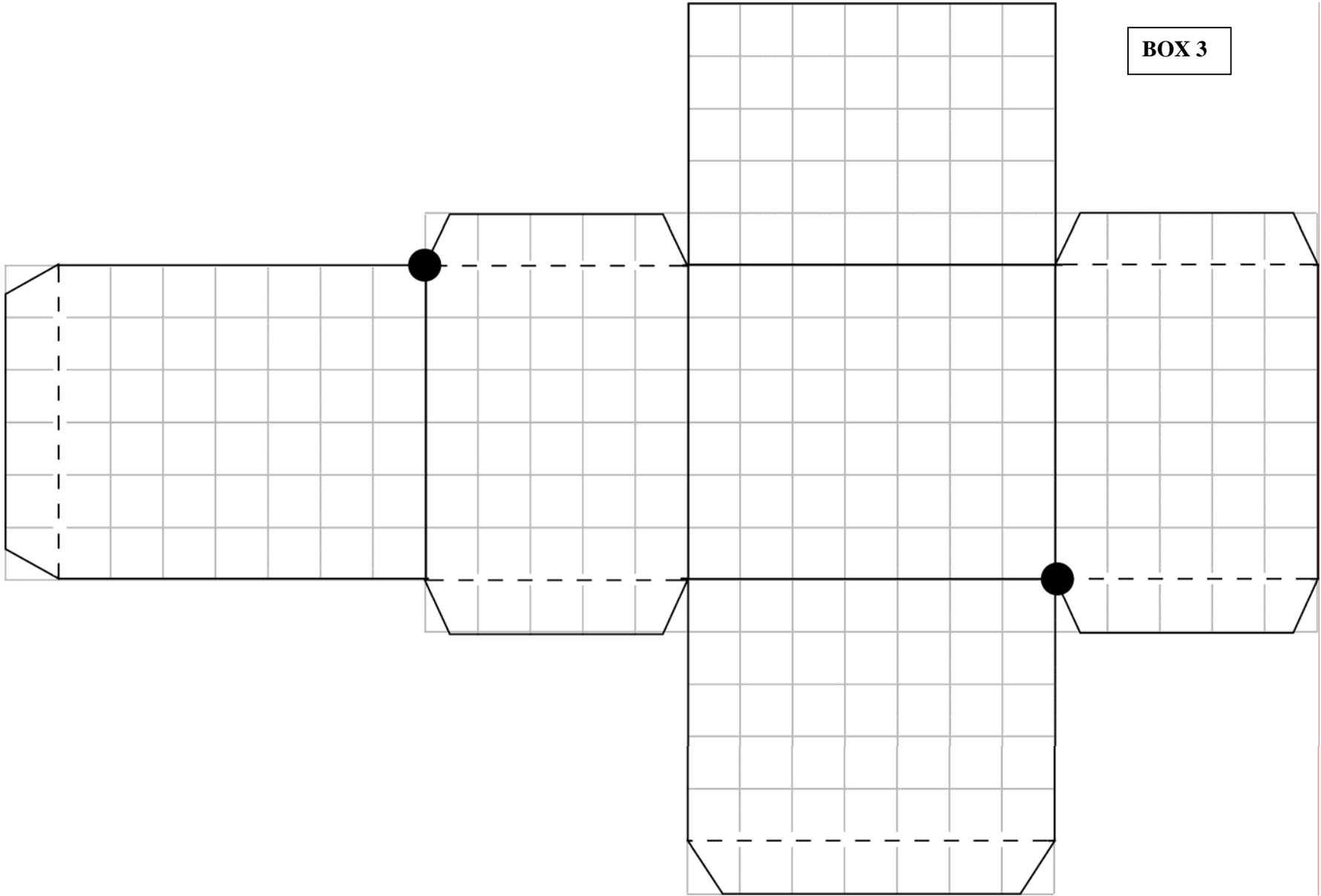


**BOX 2**

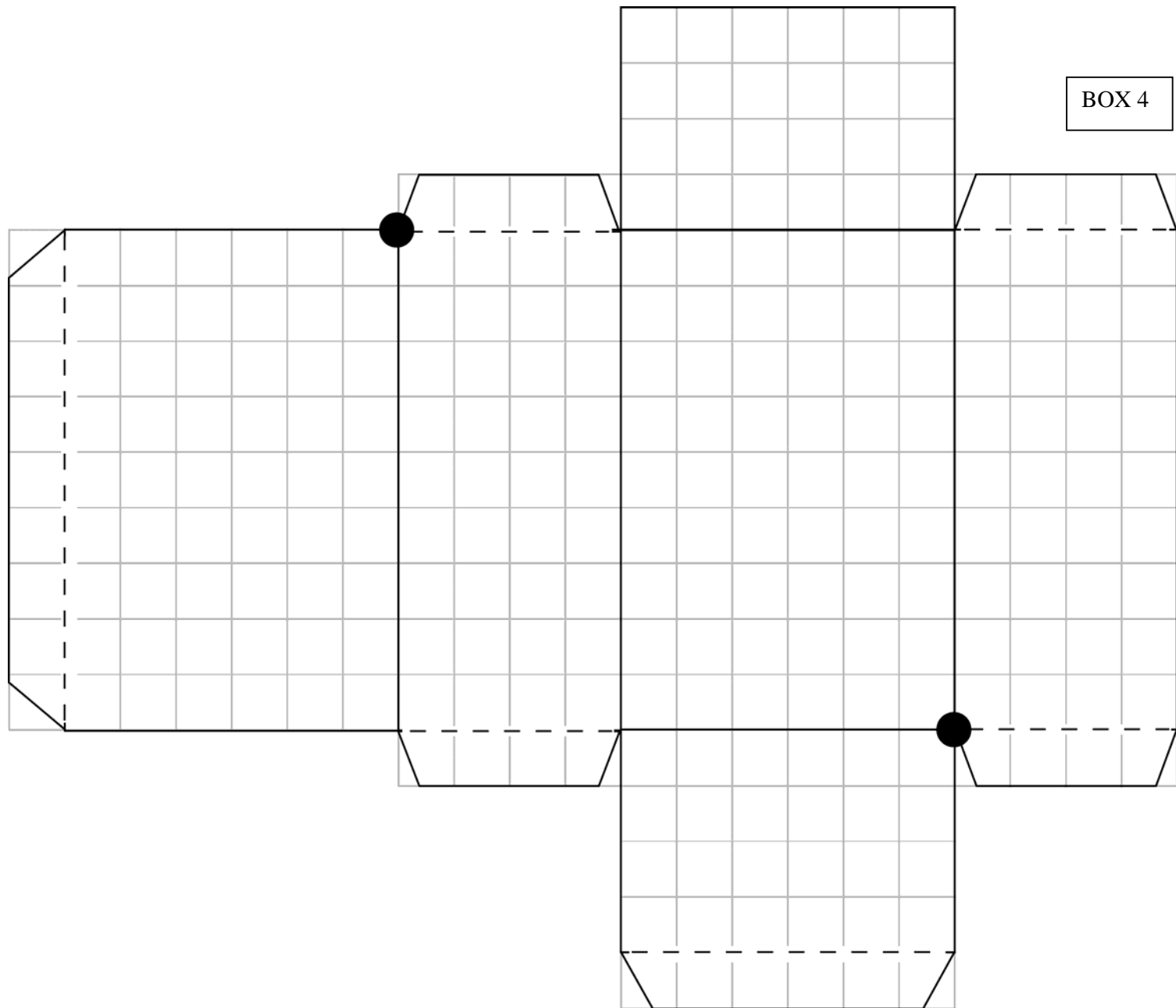




BOX 3

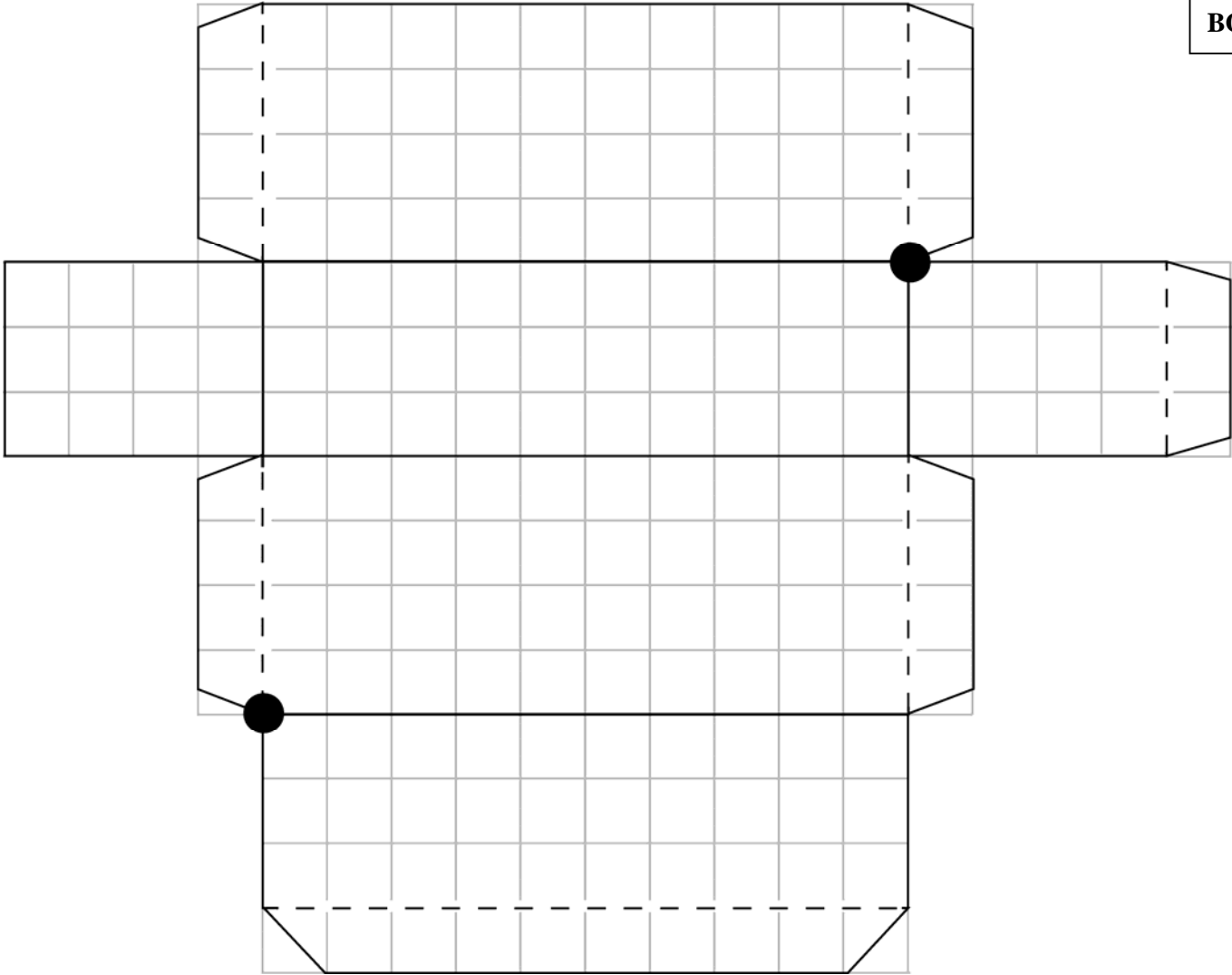






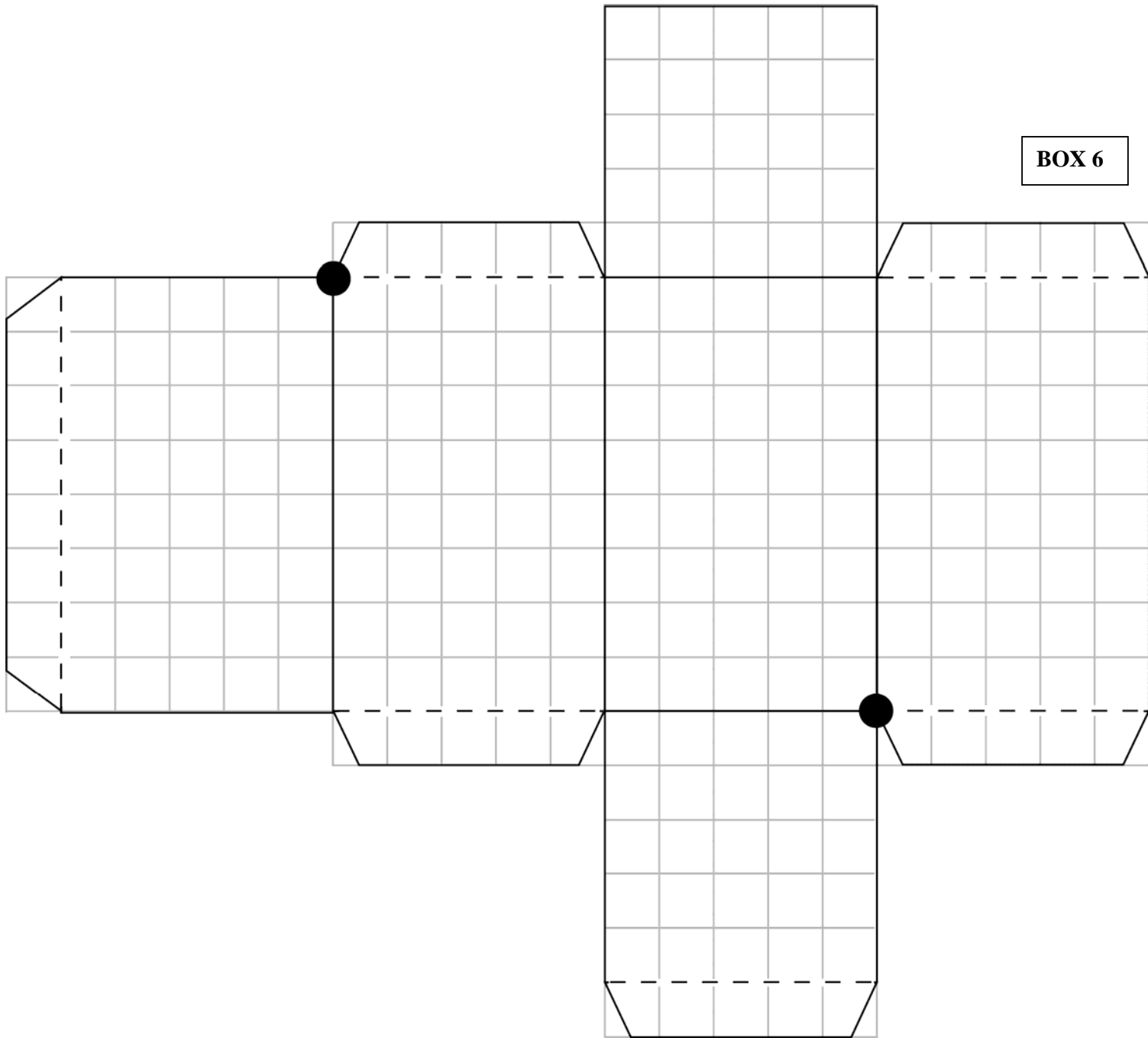


**BOX 5**









**BOX 6**

